

Solutions - Homework 1

(Due date: September 19th (001), September 18th (006) @ 5:30 pm)
Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (27 PTS)

- a) Simplify the following functions using ONLY Boolean Algebra Theorems. For each resulting simplified function, sketch the logic circuit using AND, OR, XOR, and NOT gates. (14 pts)

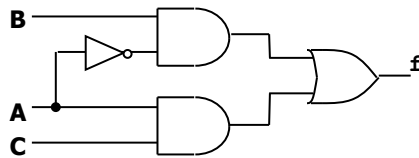
✓ $F(A, B, C) = \prod(M_0, M_1, M_4, M_6)$

✓ $F = (A + \bar{B} + D)(\bar{A}B + \bar{D})$

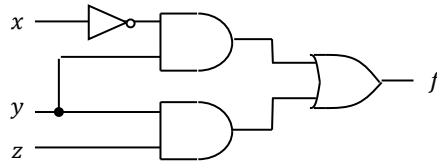
✓ $F = \overline{x(y \oplus z)} + \bar{y}$

✓ $F = \overline{(A + B)C} + AB\bar{D}$

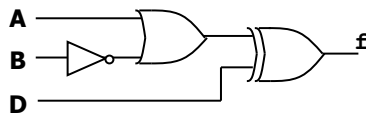
✓ $F(A, B, C) = \prod(M_0, M_1, M_4, M_6) = \sum(m_2, m_3, m_5, m_7) = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC = \bar{A}B(\bar{C} + C) + AC(\bar{B} + B) = \bar{A}B + AC$



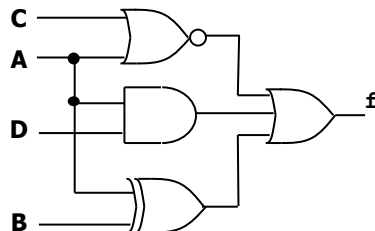
✓ $F = \overline{x(y \oplus z)} + \bar{y} = \overline{x(y \oplus z)} \cdot y = (\bar{x} + \overline{y \oplus z})y = (\bar{x} + yz + \bar{y}\bar{z})y = \bar{x}y + yz$



✓ $F = (A + \bar{B} + D)(\bar{A}B + \bar{D}) = (X + D)(\bar{X} + \bar{D}) = X\bar{D} + \bar{X}D, X = A + \bar{B}$
 $= (A + \bar{B})\bar{D} + \bar{A}BD = A\bar{D} + \bar{B}\bar{D} + \bar{A}BD = \bar{D}(A + \bar{B}) + D\bar{A}B$



✓ $F = \overline{(A + B)C} + AB\bar{D} = \overline{(A + B)C} \cdot \overline{ABD} = \bar{A}\bar{B}\bar{C} \cdot \bar{A}\bar{B}\bar{D} = (A + B + \bar{C})(\bar{A} + \bar{B} + D)$
 $= A\bar{A} + A\bar{B} + AD + B\bar{A} + B\bar{B} + BD + \bar{C}\bar{A} + \bar{C}\bar{B} + \bar{C}D = A\bar{B} + \bar{A}\bar{C} + \bar{C}\bar{B} + AD + \bar{A}B + BD + \bar{C}D$
 $= A\bar{B} + \bar{A}\bar{C} + AD + \bar{A}B + \bar{C}D = A\bar{B} + \bar{A}B + AD + \bar{A}\bar{C} + \bar{C}D = A \oplus B + AD + \bar{A}\bar{C}$



- b) Determine whether or not the following expression is valid, i.e., whether the left- and right-hand sides represent the same function. Suggestion: complete the truth tables for both sides: (5 pts)

$$x_1x_3 + \overline{x_2}\overline{x_3} + \overline{x_1}x_2 = x_2x_3 + \overline{x_1}\overline{x_3} + x_1\overline{x_2}$$

Left-hand side:

$$x_1(x_2 + \overline{x_2})x_3 + (x_1 + \overline{x_1})\overline{x_2}\overline{x_3} + \overline{x_1}x_2(x_3 + \overline{x_3}) = x_1x_2x_3 + x_1\overline{x_2}x_3 + x_1\overline{x_2}\overline{x_3} + \overline{x_1}\overline{x_2}\overline{x_3} + \overline{x_1}x_2x_3 + \overline{x_1}x_2\overline{x_3} \\ = \sum m(0,2,3,4,5,7)$$

Right-hand side:

$$(x_1 + \overline{x_1})x_2x_3 + \overline{x_1}(x_2 + \overline{x_2})\overline{x_3} + x_1\overline{x_2}(x_3 + \overline{x_3}) = x_1x_2x_3 + \overline{x_1}x_2x_3 + \overline{x_1}\overline{x_2}\overline{x_3} + \overline{x_1}\overline{x_2}x_3 + x_1\overline{x_2}x_3 + x_1\overline{x_2}\overline{x_3} \\ = \sum m(0,2,3,4,5,7)$$

Both left-hand and right-hand equations represent the same Boolean function.

- c) For the following Truth table with two outputs: (8 pts)

- Provide the Boolean functions using the Canonical Sum of Products (SOP), and Product of Sums (POS). (4 pts)
- Express the Boolean functions using the minterms and maxterms representations.
- Sketch the logic circuits as Canonical Sum of Products and Product of Sums.

x	y	z	f ₁	f ₂
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
0	1	1	0	0
1	0	0	0	1
1	0	1	1	0
1	1	0	0	0
1	1	1	1	0

Sum of Products

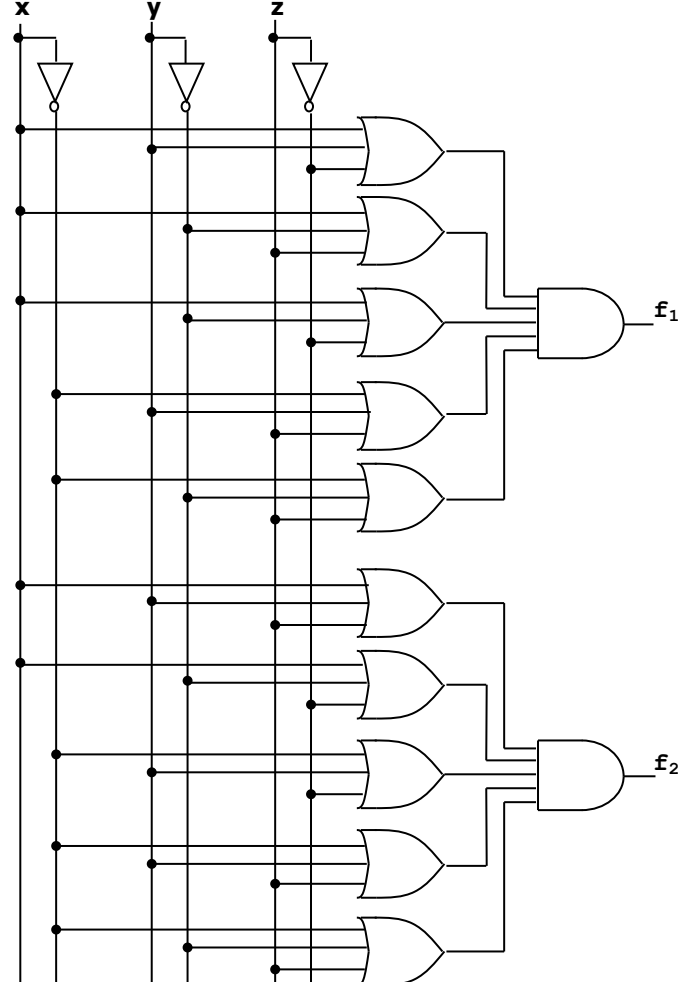
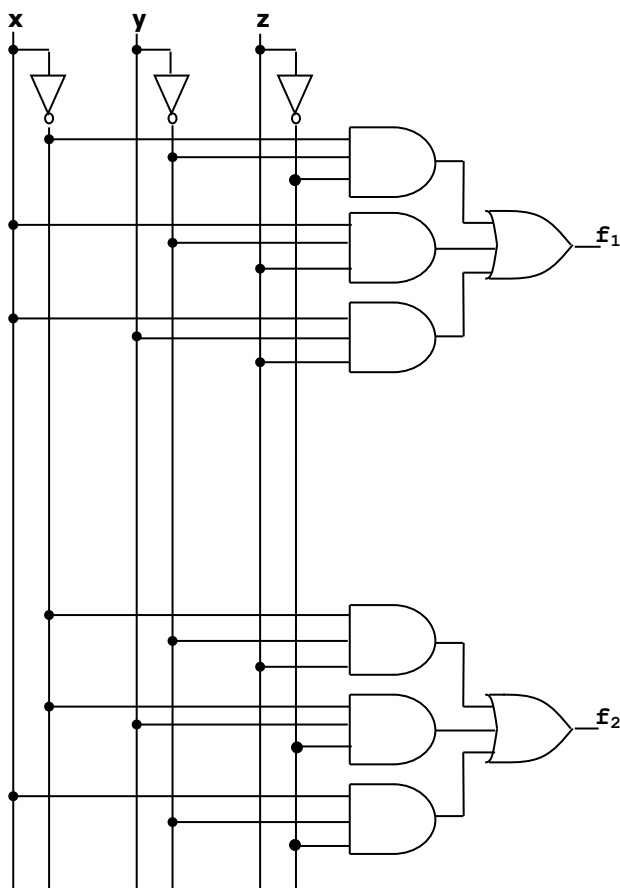
$$f_1 = \overline{x}\overline{y}\overline{z} + x\overline{y}z + xy\overline{z} \\ f_2 = \overline{x}\overline{y}z + \overline{x}y\overline{z} + x\overline{y}\overline{z}$$

Product of Sums

$$f_1 = (x + y + \overline{z})(x + \overline{y} + z)(\overline{x} + \overline{y} + \overline{z})(\overline{x} + y + z)(\overline{x} + \overline{y} + z) \\ f_2 = (x + y + z)(x + \overline{y} + \overline{z})(\overline{x} + y + \overline{z})(\overline{x} + \overline{y} + z)(\overline{x} + \overline{y} + \overline{z})$$

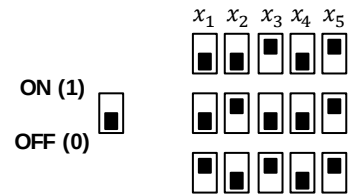
Minterms and maxterms:

$$f_1 = \sum(m_0, m_5, m_7) = \prod(M_1, M_2, M_3, M_4, M_6). \\ f_2 = \sum(m_1, m_2, m_4) = \prod(M_0, M_3, M_5, M_6, M_7).$$



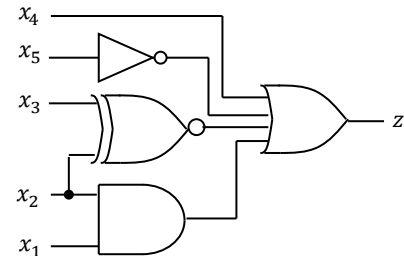
PROBLEM 2 (18 PTS)

- a) Security combinations: A lock only opens ($z = 0$) when the 5 switches (x_1, x_2, x_3, x_4, x_5) are set in any of the 3 configurations shown in the figure, otherwise the lock is closed ($z = 1$). A switch generates a '1' in the ON position, and a '0' in the OFF position.



x_1	x_2	x_3	x_4	x_5	z
0	0	1	0	1	0
0	1	0	0	1	0
1	0	1	0	1	0
All remaining cases					1

$x_1 x_2$	x_3	00	01	11	10
0	0	1	0	1	1
1	0	0	1	1	0



$$\bar{z} = \bar{x}_1 \bar{x}_2 x_3 \bar{x}_4 x_5 + \bar{x}_1 x_2 \bar{x}_3 \bar{x}_4 x_5 + x_1 \bar{x}_2 x_3 \bar{x}_4 x_5 = \bar{x}_4 x_5 (\bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 x_3)$$

$$\bar{z} = \bar{x}_4 x_5 \cdot f(x_1, x_2, x_3)$$

$$* f(x_1, x_2, x_3) = \sum m(1,2,5) \rightarrow \bar{f}(x_1, x_2, x_3) = \sum m(0,3,4,6,7)$$

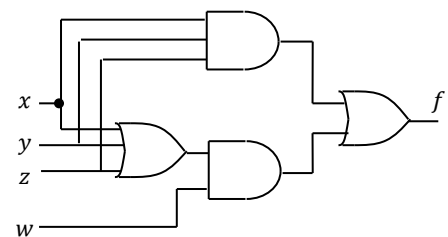
$$z = \bar{x}_4 x_5 \cdot f(x_1, x_2, x_3) = \bar{x}_4 x_5 + \bar{f}(x_1, x_2, x_3) = x_4 + \bar{x}_5 + x_2 x_3 + \bar{x}_2 \bar{x}_3 + x_1 x_2$$

- b) A doctoral student is defending his Dissertation. A 4-member committee determines whether to accept or reject the work. A simple majority vote is required. In case of a tie, the outcome is determined by the vote of the chair of the committee.
- Design the circuit (provide the simplified Boolean equation and sketch the logic circuit) that generates $f = 1$ if the committee accepts the work, and $f = 0$ if the work is rejected. We assign x, y, z, w to the vote of each committee member (w is the vote of the chair of the committee), where '1' means accept, and '0' reject. (8 pts)

x	y	z	w	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

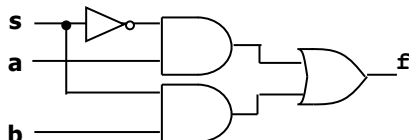
xy	zw	00	01	11	10
00	0	0	0	0	0
01	0	1	1	1	1
11	1	1	1	1	1
10	0	0	1	0	0

$$f = yw + xw + zw + xyz$$



PROBLEM 3 (13 PTS)

- a) The following circuit has the following logic function: $f = \bar{s}a + sb$.
- Complete the truth table of the circuit, and sketch the logic circuit (3 pts)

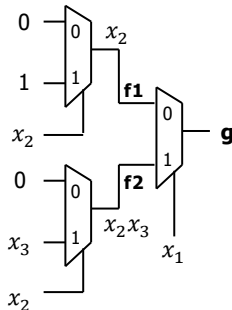


s	a	b	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

b) We can use several instances of the previous circuit to implement different functions. (10 pts)

- For example, the following selection of inputs produce the function: $g = \overline{x_1}x_2 + x_2x_3$. Demonstrate that this is the case.

in1	in2	in3	in4	in5	in6	in7
0	1	x_2	0	x_3	x_2	x_1

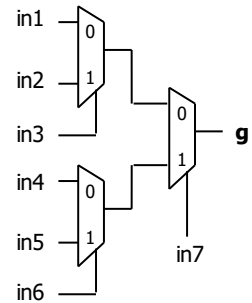


$$f_1 = \overline{x_2}(0) + x_2(1) = x_2$$

$$f_2 = \overline{x_2}(0) + x_2(x_3) = x_2x_3$$

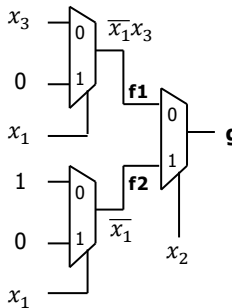
$$g = \overline{x_1}(x_2) + x_1(x_2x_3) = x_2(\overline{x_1} + x_1x_3) = x_2(\overline{x_1} + x_1)(\overline{x_1} + x_3)$$

$$g = x_2(\overline{x_1} + x_3) = \overline{x_1}x_2 + x_2x_3$$



- Given the following inputs, provide the resulting function g (minimize the function).

in1	in2	in3	in4	in5	in6	in7
x_3	0	x_1	1	0	x_1	x_2



$$f_1 = \overline{x_1}(x_3) + x_1(0) = \overline{x_1}(x_3)$$

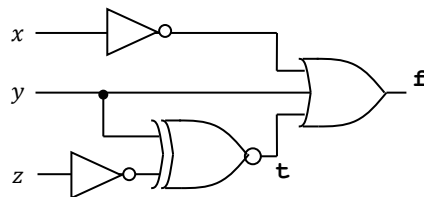
$$f_2 = \overline{x_1}(1) + x_3(0) = \overline{x_1}$$

$$g = \overline{x_2}(\overline{x_1}x_3) + x_2(\overline{x_1}) = \overline{x_1}(\overline{x_2}x_3 + x_2) = \overline{x_1}(x_2 + \overline{x_2})(x_2 + x_3)$$

$$g = \overline{x_1}x_2 + \overline{x_1}x_3$$

PROBLEM 4 (24 PTS)

a) Complete the truth table describing the output of the following circuit and write the simplified Boolean equation (6 pts).



x	y	z	t	f
0	0	0	0	1
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	0	1

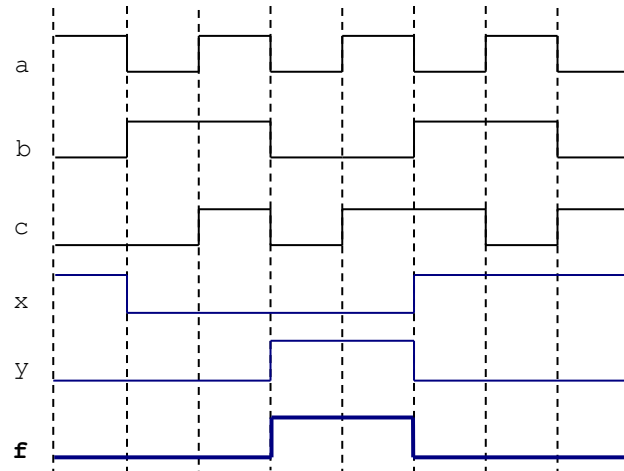
$$f = \bar{x} + y + (y \oplus z) = \bar{x} + y + z$$

b) Complete the timing diagram of the logic circuit whose VHDL description is shown below: (5 pts)

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
  port ( a, b, c: in std_logic;
        f: out std_logic);
end circ;

architecture struct of circ is
  signal x, y: std_logic;
begin
  x <= not(a) xor not(c);
  f <= y and (not b);
  y <= x nor b;
end struct;
```

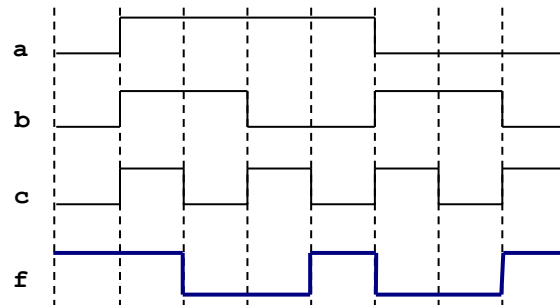


c) The following is the timing diagram of a logic circuit with 3 inputs. Sketch the logic circuit that generates this waveform. Then, complete the VHDL code. (8 pts)

```
library ieee;
use ieee.std_logic_1164.all;

entity wave is
  port ( a, b, c: in std_logic;
        f: out std_logic);
end wave;

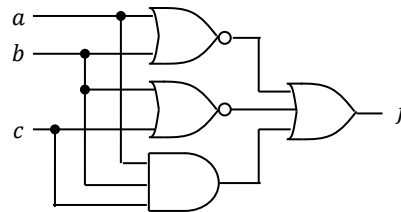
architecture struct of wave is
  signal x: std_logic;
begin
  x <= (a and b and c) or (a nor b);
  f <= (b nor c) or x;
end struct;
```



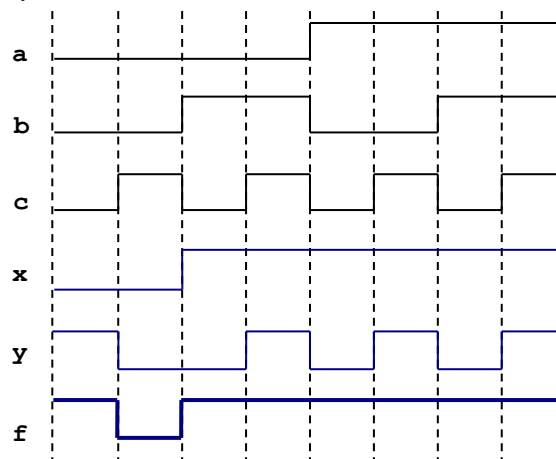
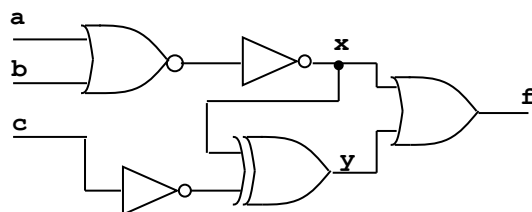
a	b	c	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

	ab			
	00	01	11	10
c 0	1	0	0	1
c 1	1	0	1	0

$$f = \bar{b}\bar{c} + \bar{a}\bar{b} + abc$$

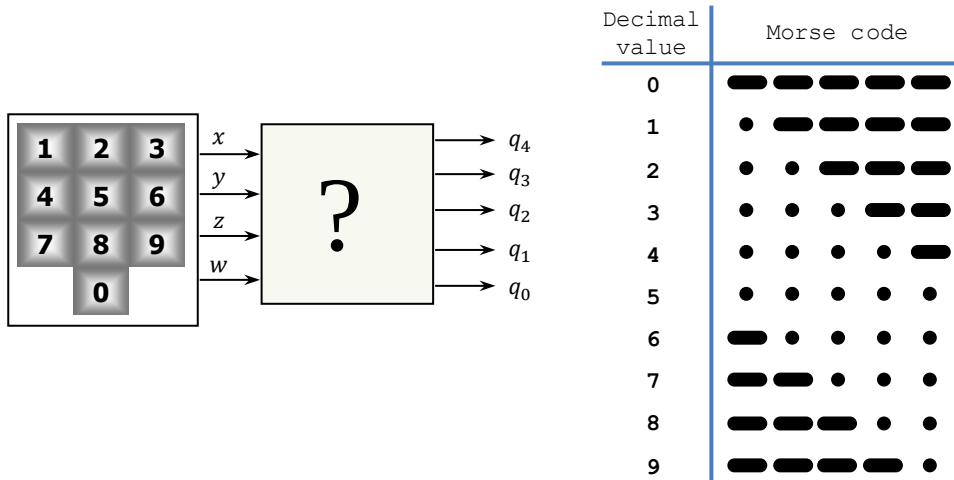


d) Complete the timing diagram of the following circuit: (5 pts)



PROBLEM 5 (18 PTS)

- A numeric keypad produces a 4-bit code $xyzw$ representing an unsigned number from 0 to 9. We want to design a logic circuit that converts each 4-bit code to Morse code (where alphanumeric characters are encoded into sequences of dots and dashes). The figure depicts the Morse code representations for numbers from 0 to 9. The circuit generates 5 bits, where a '0' represents a dot, and '1' represents a dash.



- ✓ Complete the truth table for each output (q_4, q_3, q_2, q_1, q_0). (3 pts)
- ✓ Provide the simplified expression for each output (q_4, q_3, q_2, q_1, q_0). Use Karnaugh maps for q_4, q_3, q_2 , and the Quine-McCluskey algorithm for q_1, q_0 . Note it is safe to assume that the codes 1010 to 1111 will not be produced by the keypad. (15 pts)

Value	x	y	z	w	q_4	q_3	q_2	q_1	q_0
0	0	0	0	0	1	1	1	1	1
1	0	0	0	1	0	1	1	1	1
2	0	0	1	0	0	0	1	1	1
3	0	0	1	1	0	0	0	1	1
4	0	1	0	0	0	0	0	0	1
5	0	1	0	1	0	0	0	0	0
6	0	1	1	0	1	0	0	0	0
7	0	1	1	1	1	1	0	0	0
8	1	0	0	0	1	1	1	0	0
9	1	0	0	1	1	1	1	1	0
	1	0	1	0	X	X	X	X	X
	1	0	1	1	X	X	X	X	X
	1	1	0	0	X	X	X	X	X
	1	1	0	1	X	X	X	X	X
	1	1	1	0	X	X	X	X	X
	1	1	1	1	X	X	X	X	X

q_4	xy	zw	00	01	11	10
00	1	0	X	1		
01	0	0	X	1		
11	0	1	X	X		
10	0	1	X	X		

q_3	xy	zw	00	01	11	10
00	1	0	X	1		
01	1	0	X	1		
11	0	1	X	X		
10	0	0	X	X		

q_2	xy	zw	00	01	11	10
00	1	0	X	1		
01	1	0	X	1		
11	0	0	X	X		
10	1	0	X	X		

$$q_4 = \bar{y}\bar{z}\bar{w} + x\bar{y} + yz$$

$$q_3 = \bar{y}\bar{z} + x\bar{y} + yzw$$

$$q_2 = \bar{y}\bar{z} + \bar{w}\bar{y} + x$$

$$q_1 = \sum m(0,1,2,3,9) + \sum d(10,11,12,13,14,15).$$

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants
0	$m_0 = 0000$ ✓	$m_{0,1} = 000-$ ✓ $m_{0,2} = 00-0$ ✓	$m_{0,1,2,3} = 00--$ $m_{0,2,1,3} = 00--$
1	$m_1 = 0001$ ✓ $m_2 = 0010$ ✓	$m_{1,3} = 00-1$ ✓ $m_{1,9} = -001$ ✓ $m_{2,3} = 001-$ ✓ $m_{2,10} = -010$ ✓	$m_{1,3,9,11} = -0-1$ $m_{1,9,3,11} = -0-1$ $m_{2,3,10,11} = -01-$ $m_{2,10,3,11} = -01-$
2	$m_3 = 0011$ ✓ $m_9 = 1001$ ✓ $m_{10} = 1010$ ✓ $m_{12} = 1100$ ✓	$m_{3,11} = -011$ ✓ $m_{9,11} = 10-1$ ✓ $m_{9,13} = 1-01$ ✓ $m_{10,11} = 101-$ ✓ $m_{10,14} = 1-10$ ✓ $m_{12,13} = 110-$ ✓ $m_{12,14} = 11-0$ ✓	$m_{9,11,13,15} = 1--1$ $m_{9,13,11,15} = 1--1$ $m_{10,11,14,15} = 1-1-$ $m_{10,14,11,15} = 1-1-$ $m_{12,13,14,15} = 11--$ $m_{12,14,13,15} = 11--$
3	$m_{11} = 1011$ ✓ $m_{13} = 1101$ ✓ $m_{14} = 1110$ ✓	$m_{11,15} = 1-11$ ✓ $m_{13,15} = 11-1$ ✓ $m_{14,15} = 111-$ ✓	
4	$m_{15} = 1111$ ✓		

$$q_1 = \bar{x}\bar{y} + \bar{y}w + \bar{y}z + xw + xz + xy$$

Prime Implicants		Minterms				
		0	1	2	3	9
$m_{0,1,2,3}$	$\bar{x}\bar{y}$	X	X	X	X	
$m_{1,3,9,11}$	$\bar{y}w$		X		X	X
$m_{2,3,10,11}$	$\bar{y}z$			X	X	
$m_{9,11,13,15}$	xw					X
$m_{10,11,14,15}$	xz					
$m_{12,13,14,15}$	xy					

$$\Rightarrow q_1 = \bar{x}\bar{y} + xw$$

$$q_0 = \sum m(0,1,2,3,4) + \sum d(10,11,12,13,14,15).$$

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants
0	$m_0 = 0000$ ✓	$m_{0,1} = 000-$ ✓ $m_{0,2} = 00-0$ ✓ $m_{0,4} = 0-00$	$m_{0,1,2,3} = 00--$ $m_{0,2,1,3} = 00--$
1	$m_1 = 0001$ ✓ $m_2 = 0010$ ✓ $m_4 = 0100$ ✓	$m_{1,3} = 00-1$ ✓ $m_{2,3} = 001-$ ✓ $m_{2,10} = -010$ ✓ $m_{4,12} = -100$	$m_{2,3,10,11} = -01-$ $m_{2,10,3,11} = -01-$
2	$m_3 = 0011$ ✓ $m_{10} = 1010$ ✓ $m_{12} = 1100$ ✓	$m_{3,11} = -011$ ✓ $m_{10,11} = 101-$ ✓ $m_{10,14} = 1-10$ ✓ $m_{12,13} = 110-$ ✓ $m_{12,14} = 11-0$ ✓	$m_{10,11,14,15} = 1-1-$ $m_{10,14,11,15} = 1-1-$ $m_{12,13,14,15} = 11--$ $m_{12,14,13,15} = 11--$
3	$m_{11} = 1011$ ✓ $m_{13} = 1101$ ✓ $m_{14} = 1110$ ✓	$m_{11,15} = 1-11$ ✓ $m_{13,15} = 11-1$ ✓ $m_{14,15} = 111-$ ✓	
4	$m_{15} = 1111$ ✓		

$$q_0 = \bar{x}\bar{z}\bar{w} + \bar{x}\bar{y} + \bar{y}z + xz + xy$$

Prime Implicants		Minterms				
		0	1	2	3	4
$m_{0,4}$	$\bar{x}\bar{z}\bar{w}$	X				X
$m_{4,12}$	$y\bar{z}\bar{w}$					X
$m_{0,1,2,3}$	$\bar{x}\bar{y}$	X	X	X	X	
$m_{2,3,10,11}$	$\bar{y}z$			X	X	
$m_{10,11,14,15}$	xz					
$m_{12,13,14,15}$	xy					

$$\Rightarrow q_0 = \bar{x}\bar{y} + \bar{x}\bar{z}\bar{w}$$